Assessment (Fall 1997)

University of Wisconsin-Madison

DEPARTMENT OF MATHEMATICS

Van Vleck Hall
480 Lincoln Drive
Madison, WI 53706-1388

Telephone: (608) 263-3053
E-Mail: recep@math.wisc.edu
Fax number: (608) 263-8891

AN

October 22, 1997

To: Alexander Nagel, Associate Dean

From: Richard A. Brualdi, Chair RAB

I am enclosing the Report on our Assessment Activities in 1996-97. This report was reviewed at our department meeting on October 16, 1997, and will be discussed and acted upon by our Undergraduate Program Committee at a meeting soon. I would be happy to discuss any concerns you may have with our assessment activities.

RAB/dz

Report

of the

1996 - 1997

Mathematics Department

Assessment Committee

This past academic year, in collaboration with course instructors, we again selected several of our undergraduate courses for assessment. For the Fall semester, we chose Math 340 and 541, and for the Spring semester 371, 431, and 521. Math 340 and 541 had been assessed during the preceding year. Our analysis of those assessments suggested the need to revisit those courses. Our spring selection was guided by an attempt to choose at least 2 courses, not previously assessed, whose success we consider important to our undergraduate program.

Math 431 is our basic probability course. It serves an increasing interest on the part of many disciplines in probability theory (besides the obvious ones of Statistics and Mathematics itself e.g. Computer Science, Economics, Business, and Engineering). Math 521 remains the launching pad to graduate level courses in analysis. We threw Math 371 in to the mix to acquire some sense of the effectiveness of that relatively new course, whose purpose is to help students seeking a bridge from sophomore level mathematics to abstract and proof intensive courses.

Our assessment procedure was virtually the same as that employed last year. For each of the courses, one problem on the final exam was designated for assessment. The particular problem to be assessed was to be of the nature of a problem which would reveal whether one (or more) of the basic goals of the course was achieved - e.g. writing a proof after a semester of 521 or 541.

Susan Hollingsworth, TA, who ably assessed last year's problems also graded for us this year. Two of the aspects of solutions or proofs that Susan rated, which were not evaluated last year, were:

(a) Good organization of a solution,

(b) clear sentences when giving explanations.

COURSE DESCRIPTIONS

MATH 340 — ELEMENTARY MATRIX AND LINEAR ALGEBRA — 3CR

This course introduces the student to matrix and linear algebra which are used in many advanced math courses and courses in other departments. Math 340 also serves as a bridge between the problem solving calculus courses and the more abstract advanced math courses; it is a prerequisite for 521, 541 and many other advanced courses. Topics: Matrix algebra, systems of linear equations, determinants, vector spaces, linear independence, bases, dimension, linear transformations, eigenvalues, eigenvectors, inner product, orthogonality, diagonalization. Possible text: Elementary Linear Algebra, 6th Ed., by H. Anton; Linear Algebra and Its Applications, by D.C. Lay. Prereq: 223. Students may not receive credit for both 320 and 340.

MATH 371 — BASIC CONCEPTS OF MATHEMATICS — 3CR

This course is designed to help students make the transition to the 500 and 600 level courses in which there is more emphasis on proofs. It will help students understand proofs and devise concise proofs as well as introduce them to some basic mathematical knowledge. Topics: informal treatment of propositional and first-order logic; proof techniques; naive set theory; relations and functions; Peano axioms; construction of the real numbers; countable and uncountable sets; Axiom of Choice and Zorn's Lemma. Prereq: Math 340 or concurrent registration in 340.

MATH 431 — INTRODUCTION TO THE THEORY OF PROBABILITY — 3CR

(Same as Statistics 431)

This is an introduction to the basic ideas of probability for students with a good calculus background. It is of particular interest to students in mathematics, statistics, physical and biological sciences, and engineering and also to students in some of the social sciences. Topics: sample spaces, probability measures, combinatorial analysis, conditional probability, independence, random variables, distributions, expectation, laws of large numbers, central limit theorem. Possible text: A First Course in Probability, 4th ed, by Ross; Probability and Stochastic Processes by Solomon. Prereq: Math 223.

MATH 521 — ADVANCED CALCULUS — 3CR

This sequence introduces students to the terminology, fundamental concepts and basic elementary theorems of analysis with emphasis on functions of several variables. The objective is to convey an understanding of the structure of analysis in itself as well as its role as a tool for other disciplines. This sequence is essential for students preparing for graduate studies in mathematics; also it should be taken by students of physics and engineering who intend to do graduate work in their areas. Topics in 521: topological notions, mappings, continuity, differentiation, integration ,series and possibly Fourier series. Possible texts: Advanced Calculus, 3rd ed. by Buck; Elementary Classical Analysis by Marsden; Elementary Classical Analysis, by Marsden; Principles of Mathematical Analysis, 3rd ed, by Rudin; Advanced Calculus, by Taylor and Mann; Undergraduate Analysis, by Lang. Prereq for 521: Math 340 or concurrent enrollment.

MATH 541 — MODERN ALGEBRA — 3CR

This is the first semester of an introduction to basic abstract algebra. It is essential for students preparing for graduate studies in mathematics or in some related fields. Topics: group theory: subgroups, homomorphisms, isomorphisms, normal subgroups, permutation groups, class equation, Sylow theorem, finite abelian groups; ring theory: homomorphisms, isomorphisms, ideals, integral domains, polynomial rings. Possible texts: Abstract Algebra by Herstein; Contemporary Abstract Algebra by Gallian. Prereq: Math 320 or 340.

PLAN FOR ASSESSING THE MATHEMATICS MAJOR

1. GOALS

The overall goal of our undergraduate mathematics major is to produce students who understand and appreciate mathematics, who can use mathematics in understanding the world, and who can use mathematics as a basis for life-long learning. Included in this overall goal is the belief that completing the major in mathematics entails gaining sufficient subject competency to enable a student to achieve at least one of the following:

a. To handle the mathematical demands of a technical entry level position in business, industry, or government.

b. To pursue a graduate program in the mathematical sciences.

c. To handle the mathematical demands in pursuing a scientific graduate or professional program.

d. To teach mathematics in a secondary school.

2. OBJECTIVES

Students completing the major in mathematics should have attained the following:

a. They should be able to use the language of mathematics both in its idiomatic and rigorous forms. They should be able to give a clear written or oral explanations of the meaning of certain fundamental concepts or statements, or of how such statements or concepts apply in a particular situation. This ability includes interpreting and using conventional mathematical notation.

b. They should have reasonable facility with the basic mathematical techniques used in a required area of study, and a knowledge of basic theorems in this area.

c. They should be able to construct simple mathematical proofs, and to formulate and test conjectures.

d. They should be able to apply what they have learned in one mathematical area to another area, whether by modeling a physical situation or interpreting one mathematical object or structure in terms of another.

3. IMPLEMENTATION

Each year the chair shall appoint an Assessment Committee which shall be charged with gathering data indicating the extent to which the department is meeting its objectives, and, where indicated, to make recommendations based on their findings. Specifically, the Assessment Committee shall

a. Each year select one or more courses central to our major program, which will be assessed. In each of these courses, the committee, together with instructors teaching the course, will identify a question which will be made part of one of the usual course examinations. Performance on this question will be used by the committee as a measure of the attainment of the department's objectives. The questions chosen will be of types normal for the courses chosen. The responses to the questions will be graded as usual by each instructor as part of the grading process. They will also be graded independently by someone hired to assist the Assessment Committee. The questions will normally be identified early in the semester by the committee and the instructors involved. The Assessment Committee will meet with participating faculty and course coordinators for the courses involved (which may include prerequisite courses) to discuss the results.

The courses selected by the committee for this process will normally be chosen from Math 441, Math 521, or Math 541, or other such courses as the committee judges will yield useful data to measure progress toward the department's objectives.

b. From time to time, conduct exit interviews or surveys with mathematics majors who are about to graduate.

c. About every five to seven years, conduct surveys of our graduates several years after they have graduated.

d. Collect data in such other ways as the Assessment Committee shall deem helpful and appropriate.

Once each year the Assessment Committee shall prepare a report on its activities for the year, and its evaluation of the outcome. It shall meet with the Undergraduate Program Committee to present its report and to discuss possible program modifications or improvements relevant to the material in its report.

REPORT OF THE UNDERGRADUATE PROGRAM COMMITTEE ON ASSESSMENT PROCEDURES

The Board of Regents and the North Central Association have mandated that the University develop procedures for measuring and evaluating student outcomes in general education, in each undergraduate major, and in graduate education. There are three major components that must be present in any assessment plan. They are (according to a L&S document on assessment):

1. Each unit that is being assessed should articulate clearly and precisely a set of EDUCATIONAL GOALS.

2. Each unit should develop MEANS OF ASSESSMENT that measure the extent to which it achieves the articulated goals.

3. Each unit should use these measurements to MONITOR ITS PROGRAM AND MAKE CHANGES. These may be changes in the methods used to reach the goals, or may be changes in the goals themselves.

SUMMARY OF RESULTS

Math 340 Last year's assessment of this course seemed to confirm what many of us suspected. A large majority of the students ended the semester without understanding the general concepts in the course. With the reinstitution of Math 320 becoming more firmly established, we decided to revisit 340 to see whether 320 may be siphoning off the type of student with little interest or motivation for comprehending the ideas and theorems of Linear Algebra. The problem this time round was different from the one we assessed last year. It required some understanding of only one idea, linear dependence, a recognition of its relevance to the question, and the ability to correctly apply the definition to justify the desired conclusion. Of the 26 students 12 seemed to recognize that linear dependence was relevant, but few were able to precisely formulate the connection.

Math 371 The selected problem in this course dealt with applying basic notions for functions - defining a desired function, its domain and range, whether it has left and right inverses, and the relevance of the properties of being one to one and onto to the question.

Most of the students seemed to understand the basic definitions, knew how to organize their arguments and present them in full sentences - among the major goals of the course. A majority recognized the need to justify assertions. Few were able to deal successfully with the questions posed in the problem which required an understanding of the somewhat less obvious general properties of mappings.

The performance on this problem seems to indicate that the course helps to prepare students for some important aspects of 500 level courses e.g. definitions and proofs. However, few of these students exhibited sufficient sophistication to gauge their potential for success in those courses.

Math 431 The first part of the problem instructs the students to apply the Central Limit Theorem to the problem at hand to obtain the required estimate. Of the 25 students 18 applied the theorem to the problem to obtain a correct answer. However, many of these students presented their solutions as if they were dealing with an equality, not an estimate.

The second part of the problem required both an application and an explanation. The application was done correctly and lucidly explained by only six of the students. Others gave apparently memorized explanations which in fact contradicted their own calculations - a troublesome phenomenon all too familiar to all of us.

Math 521 This three part problem asks for a correct statement of a standard test for uniform convergence of a series. It then presents a specific series and requires a determination of the sets of pointwise convergence and of uniform convergence.

The problems tests for 3 of the goals of the course - writing a precise statement of a theorem, understanding concepts, and knowing how to apply a theorem to a specific example.

Of the 17 students in this class about half of the students presented an attempted statement of the theorem clearly. None of the students had a completely correct statement, although 4 came close. A majority of the student were able to determine the set of point-wise convergence with an adequate justification. Only 7 of the 17 could correctly determine where the given series is uniformly convergent.

Math 541 This too is a revisited course. We noted in our assessment report last year that the then designated problem tested the ability to recall and clearly state a particular definition and to apply it in a calculation. It did not examine the ability to provide a proof of a correct assertion - a primary goal of this course. The selected problem this year required a proof of a given statement.

We note first that of 24 students ordinally enrolled, only 13 survived to take the final exam. Most of these 13 students did quite well on this problem and seem to have learned how to present a proof.

Exam Questions

Math 340 Question

Let A be an $n \times n$ matrix such that the sum of the entries in each of its columns is 0. What can you say about Det$A$? Justify your answer.

Math 371 Question

A function $g : B \rightarrow A$ is a left inverse for $f : A \rightarrow B$ iff $g \circ f = I_A$, and a right inverse iff $f \circ g = I_B$. (Here I_X denotes the identity function of the set X.) The function g is an inverse to f if it is both a left inverse and a right inverse.

1. Give an example of a left inverse which is not a right inverse.
2. Which functions have a left inverse? Which have a right inverse?
3. Let $f : \{0, 1, 2, 3, 4, 5, 6\} \rightarrow \{0, 1, 2\}$ be defined by the condition that $f(x)$ is the remainder when x is divided by 3, i.e. $f(1) - f(4) = 1$ etc. How many right inverses does f have? How many left inverses?
4. Suppose $f : A \rightarrow A$ has a right inverse. Does it have a left inverse? What if A is finite?
5. Suppose that g_1 and g_2 are left inverses for f and h is a right inverse for f. Must $g_1 = g_2$?

Math 431 Question

A particle moves on the integers $\{0, \pm 1, \pm 2, \ldots\}$. It starts at 0 and moves one unit to the right, or one to the left, or stays put, each with probability 1/3. Let S_n = its location after n moves.

(a) Use the Central Limit Theorem to estimate $P\{-6 \leq S_{54} \leq 6\}$.

(b) What does the Chebycheff inequality tell you about this probability? Which estimate is more informative? Why?

Math 521 Question

(a) Give a precise statement of the Weierstrass M-test for the uniform convergence of a series of functions.

(b) For which values of x does the series $\sum_{k=0}^{\infty} e^{-kx}$ converge?

(c) For which intervals I does the series $\sum_{k=0}^{\infty} e^{-kx}$ converge uniformly for $x \in I$?

Math 541 Question

Let G denote a finite group of order $p \cdot q$, where p and q are prime. Show that G has a proper normal sub-group.

Grader

Comments

and

Scores

340 (26 papers)

Understanding of the problem

Understanding of hypothesis (columns all zero) Almost all the students demonstrated that they understood the hypothesis of the problem. A few misinterpreted it as the *row* sums being zero, or the row *and* column sums being zero. One thought that the sum of the first $n-1$ entries of each column was zero.

Understanding of what was asked (say something about $\det A$**)** Again, almost everyone understood that they had to make some statement about $\det A$. A few students interpreted this to mean that they could make some sort of obvious claim, such as $\det A = \det A^T$, or that $\det A$ depends on the entries of A.

understood hypothesis	yes	20
	sorta	3
	no	1
	can't tell	2
understood what was asked	yes	23
	sorta	3
	no	0
	can't tell	0

Organization

Clear Progression I evaluated how easy it was to follow the sequence of steps in the student's argument. Overall the arguments were fairly disorganized, although a large proportion of the students did organize their work so it was easy to follow. I scored as follows:

0 utter chaos

1 mostly random, but can be followed with effort

2 lost of randomly placed statements, but some parts are organized

3 mostly organized, a few randomly placed statements

4 easy to follow

Extraneous Statements I marked whether the student left extraneous statements not necessary to the argument, *e.g.* scratch work or irrelevant remarks. Most of the students left some scratch work (examples with no explanatory remarks), but none had remarks or statements irrelevant to the problem.

Use of Full Sentences I evaluated the student's use of full sentences (both English and symbolic) in the argument. Almost all the students had some mixture of full sentences and garbled fragments.

Justification I noted how the student justified his or her statements:

- **0** no justification
- **1** only by specific examples
- **2** by matrices with generic entries, but only small size (2×2 and 3×3)
- **3** justification in full generality
- **n/a** no statements to justify!

clear progression	0 — chaos	1
	1	4
	2	6
	3	4
	4 — clear	11
extraneous statements	some	15
	none	11
full sentences	not at all	0
	mostly not	4
	sometimes	8
	mostly so	6
	always	8
justification	0 — none	3
	1 — specific	11
	2 — small	7
	3 — general	3
	n/a	2

Overall Correctness of Argument I scored from 0 (pure junk) to 4 (perfectly correct).

pure junk	0	12
	1	2
	2	3
	3	5
perfectly correct	4	4

371 (27 papers)

An example of a left inverse which is not a right inverse

Defined a function f Almost all of the students did explicitly define a function which they claimed possessed a left inverse but not a right, thereby indicating that they understood that the problem required them to exhibit an example.

Defined domain, range of f About three-quarters of the students who defined the function also defined the domain and range.

Defined a function g Fewer of the students realized that they needed to define the purported left-but-not-right inverse. Three of the students claimed that the function g exists, but did not exhibit it.

The function g is in fact a left inverse Just over half of the students succeeded in choosing a function g which was in fact a left inverse for their particular function f.

Showed that g is a left inverse Only about a third of the students considered it necessary to show that the function g they had defined was in fact a left inverse for their function f.

The function g is in fact not a right inverse Because some students defined a function g which was in fact a two-sided inverse for their particular f, only 11 of the 27 students succeeded in this category.

Showed that g is not a right inverse Only eight students explicitly demonstrated that their particular g was not a right inverse for their particular f. However, two more students claimed this was true without explicit demonstrations by citing arguments about 1-1 functions.

Two different students said [in paraphrase], "Here is a function which is 1-1 but not onto, so it *is* a left inverse but not a right."

defined function f	22
defined domain, range of f	19
defined function g	17
g is in fact left inverse	15
showed that g is a left inverse	9
g is not a right inverse	11
showed that g is not a right inverse	8

When functions possess left, right inverses

Twenty-three of the twenty-seven students knew that 1-1 functions have left inverses and onto functions have right inverses. However, only *two* students gave a proof, and one other said "the proof was given in class." The rest simply stated the fact.

Counting left, right inverses

Almost all the student (25 of 27) recognized that the function had no left inverses, and of those, 20 were able to say why.

The count of right inverses involved invoking a formula presumably explained in class. Twenty of the students correctly counted the number of right inverses, but only nine gave any sort of reasonable explanation. Many gave no explanation at all, and many others gave explanations which made little to no sense. Here are some examples of these:

- "f is onto in 12 cases, because $3 \times 2 \times 2 = 12$."
- "there are 3-0's, 2-1's, and 2-2's, so $3 \times 2 \times 2 = 12$."
- "only 12 functions will satisfy ontoness"
- "3 functions $\rightarrow$ 0, 2 functions $\rightarrow$ 1, 2 functions $\rightarrow$ 2, therefore $3 \times 2 \times 2 = 12$."
- "$3 \times 2 \times 2$" appeared somewhere in the answer but lacking any other explanation

Matching cardinalities and inverses

Only eight of the students gave an example to illustrate the general case. I saw a number of variations on "no, because it may be infinite" as an attempt at the general case. Some students observed that in the finite case the function must be 1-1 but never explicitly stated that this would imply the existence of a left inverse.

Two students had "maybe" as the answer for the general case, but made no attempt to explain. Three students had "yes" as the answer for the finite case, but made no attempt to explain.

Four students said "f has a left inverse iff f is 1-1" for both the general and the finite cases.

I scaled in three categories:

Organization Overall the arguments were reasonably well organized. I scored as follows:

0 utter chaos

1 mostly random, but can be followed with effort

2 lost of randomly placed statements, but some parts are organized

3 mostly organized, a few randomly placed statements

4 easy to follow

Use of full sentences I looked for the use of full sentences, both English and symbolic. On the whole the students did this reasonably well. Again I scored from 0 to 4.

Correctness of argument I scored separately for the general and the finite cases. Many students did not know how to approach the general case.

	0	1	2	3	4
organization	1	1	4	4	17
full sentences	1	1	4	7	14
general case	14	0	2	3	8
finite case	8	5	5	4	5

Unique inverses

One student left this part blank, so I scored only 26 papers.

I began by scoring for style.

Organization Overall the arguments were reasonably well organized. I scored as follows:

0 utter chaos

1 mostly random, but can be followed with effort

2 lost of randomly placed statements, but some parts are organized

3 mostly organized, a few randomly placed statements

4 easy to follow

Use of full sentences I looked for the use of full sentences, both English and symbolic. On the whole the students did this reasonably well. Again I scored from 0 to 4.

	0	1	2	3	4
organization	1	2	5	9	9
full sentences	0	3	9	5	9

Next I looked at the correctness of the argument. I was not impressed. Only seven students gave a full proof, and of these, five used notation without defining it. Twelve more students made some variation on the statement "f is a bijection, so it has a unique inverse," but without providing additional explanation. The rest of the answers were basically nonsensical:

- "$g_1 = g_2$ (if and) only if f is also 1-1 and onto."
- "If g_1 and g_2 are left inverses for f, then they all have the same number of elements (1:1) h is onto therefore yes, $g_1 = g_2$."
- "g_1 must equal g_2 if f is one-to-one and onto for every left inverse = every right inverse = every other left inverse."
- "yes because a left + right inverse is present. Therefore f has an inverse."
- "No f can have multiple left inverses."
- "Yes, because the function is 1-1 and onto so they have the same number of elements."

431 (25 papers)

CLT

It should be noted that only six of the students indicated that the answer they came up with was an approximation.

Defined notation A lot of students used notation without defining it first. I scored as follows:

0 almost no symbols are defined

1 liberally peppered with undefined symbols

2 some undefined symbols, but most defined

3 only one or two undefined symbols

4 all notation defined

Organization I evaluated how easy it was to follow the sequence of steps in the student's argument. I graded as follows:

0 utter chaos

1 mostly random, but can be followed with effort

2 lost of randomly placed statements, but some parts are organized

3 mostly organized, a few randomly placed statements

4 easy to follow

Use of full sentences I looked for full sentences, both English and symbolic.

Intermediate computations I noted whether the student attempted to make useful intermediate computations, and also whether these computations were correctly performed.

Applied CLT Here I tried to grade the student's ability to actually apply the CLT to this problem.

	0	1	2	3	4
defined notation	6	3	7	3	5
organization	1	7	8	7	2
sentences	0	2	5	8	10
useful computations	1	1	4	0	19
correct computations	2	3	1	1	18
applied CLT	1	2	1	1	20

Chebycheff

Correctly set up Chebycheff I evaluated how well the student set up the calculation.

Lucidity of explanation This category mostly involves use of English, but also includes logical structure of the answer and general writing style. Although some students did a good job, on the whole the explanations were fair to poor.

	0	1	2	3	4
setup Cheby	3	0	6	8	6
lucidity	3	2	6	6	7

I suspect a lot of "regurgitating" went on in this problem, because a lot of students computed the inequality incorrectly but then "explained" that Chebycheff is in general not as good (even if this contradicted their own answer).

correct inequality, correct explanation	6
incorrect inequality, "correct" explanation (parrot?)	8
incorrect inequality, nonsensical explanation	9
incorrect inequality, reasonable explanation	1

521 (17 papers)

Definition

Sentences with logical structure I looked for complete, well-constructed English and symbolic sentences. I used high standards because the problem asked for a *statement* of the M-test, which the student should understand to mean should consist exclusively of meaningful sentences.

Correctness of statement I looked for completeness and accuracy of the description of the M-test. There was some double-jeopardy because of overlap with the "full sentences" category above: if the sentences were not well-phrased, I did not consider the test correctly stated.

Defined notation I noticed that there was a lot of undefined notation floating around, so I created a category for this, as well. I scored as follows:

0 almost no symbols are defined

1 liberally peppered with undefined symbols

2 some undefined symbols, but most defined

3 only one or two undefined symbols

4 all notation defined

	0	1	2	3	4
sentences	2	2	4	7	2
correctness	7	2	3	4	0
defined notation	0	3	3	5	6

Interval of convergence

Organization I evaluated how easy it was to follow the sequence of steps in the student's argument. I graded as follows:

0 utter chaos

1 mostly random, but can be followed with effort

2 lost of randomly placed statements, but some parts are organized

3 mostly organized, a few randomly placed statements

4 easy to follow

Use of full sentences I looked for full sentences, both English and symbolic.

Explicitly answered question Some students did not explicitly give a range of numbers as the question demanded, although they may have gone through the calculations required.

Examined all possibilities I noted whether they examined numbers positive, negative, and zero. About half the students neglected at least one of these.

Cited test being used I noted whether they explicitly named the test they were using.

Correct use of test I scored for the correct use of the test they intended to use.

	0	1	2	3	4
organization	2	1	2	3	9
sentences	2	1	2	7	5
explicit answer	1	0	2	3	11
examined possibilities	3	2	2	0	10
cited test	3	3	2	0	9
use of test	7	1	2	3	4

Use of M-test

I used the same categories as above. Notice that here, very few students examined all possibilities (positive, negative, and zero) and over half neglected to cite the test being used.

	0	1	2	3	4
organization	4	2	3	3	5
sentences	3	1	1	6	6
explicit answer	1	3	3	5	5
examined possibilities	13	2	0	0	2
cited test	9	0	0	0	8
use of test	9	1	1	0	6

541 (13 papers)

Understanding of hypothesis Virtually all the students apparently understood the hypotheses of the problem.

Understanding of what was asked Again, virtually all understood what the problem asked for. One student showed that the trivial subgroup is normal, possibly because the original problem statement didn't ask for a *proper* normal subgroup (but mostly because he was one confused puppy).

understood hypothesis	yes	12
	sorta	0
	no	0
	can't tell	1
understood what was asked	yes	11
	sorta	0
	no	1
	can't tell	1

Organization

Clear Progression This group of students did a good job overall of organizing their arguments in an easily-understood manner. I used the same coding system as for math 340.

Extraneous Statements A fair number of students had remarks not useful to their argument.

Use of Full Sentences On the whole the students used full sentences (English and symbolic) in their work.

Justification Most of the students justified their statements. It is worth noting that only three of the thirteen mentioned the Sylow theorems by name. Unlike the math 340 students, none of the 541 students used specific examples as supporting evidence. I coded as follows:

0 no statements are justified

1 many statements are unjustified

2 most statements are justified

3 all statements are justified

Remark on Notation Exactly one student used some non-standard notation without explaining what was meant by it. I would have expected this to be more common, so I thought it was worthy of note.

clear progression	0 — chaos	0
	1	0
	2	2
	3	2
	4 — clear	9
extraneous statements	some	7
	none	6
full sentences	not at all	0
	mostly not	0
	sometimes	1
	mostly so	2
	always	10
justification	0 — none	1
	1 — some	3
	2 — mostly	1
	3 — fully	8

Overall Correctness of Argument I scored from 0 (pure junk) to 4 (perfectly correct).

pure junk	0	2
	1	0
	2	3
	3	2
perfectly correct	4	6

SOME GENERAL OBSERVATIONS BY THE COMMITTEE CHAIR

Math 340 - Linear Algebra In the 1995-96 assessment of this course we noted that a majority of the students failed to grasp the general concepts. We find that again to be the case in our reassessment. With the reinstatement of Math 320 we now have a suitable Linear Algebra alternative for engineering students. Since a course in linear algebra may take many forms, the department may wish to consider the additional alternative of a matrix oriented course which may better serve non-engineering students who do not plan on 500 level mathematics courses, e.g. Computer Science, Economics, and Business majors. We should attempt to steer into our Math 340 course only those students who have the potential to deal effectively with the concepts and who may intend a program involving courses at the 500 level.

Math 371 -Basic Concepts of Mathematics Does this relatively new offering serve its intended purpose of preparing students for "theorem-proof" type courses? The results of the assessment problem do not give a conclusive answer. The department should compare the performance in 500 level courses of students with comparable math grades who have not taken 371 with those who have, to get a better sense of the effectiveness of Math 371.

Math 431 - Elementary Probability Theory Student performance on the assessment problem reinforces our concern that all too many students rely on applying results without a clear understanding of their content. How do we get our students to understand the mathematics they're using, rather than just relying on memorized methods applied without comprehension? This difficulty is pervasive in many of our courses.

Math 521 - Advanced Calculus Even after a full semester of Advanced Calculus many students have difficulty writing a clear mathematical statement. Math 521 is a real hurdle for most students who attempt it. Shouldn't we be encouraging many of these students to take 321 instead of 521? If they are not contemplating a graduate degree in mathematics they may benefit more from 321.

Math 541 - Abstract Algebra The 13 students who took the final exam are the survivors from an initial group of 24 students registered for the course at the beginning of the semester. On the whole, these survivors handled the assessment problem rather well. Is the moral here that both faculty and students would be better served if weaker students could be discouraged from attempting this course?

Preliminary report on the Wisconsin Emerging Scholars (WES) Program

1. WES began in the Fall of the '93-'94 academic year, with Mike Bleicher and Melinda Certain serving as "TA's" for two sections of Math 221.

2. This semester there are five sections: two of 221, two of 222, and one of 223. Typically there are four or five sections each semester.

3. The main feature is a "workshop" approach to the discussion section of calculus, in which students work collaboratively in small groups on a worksheet of problems prepared by the TA. Workshops meet three times a week for two hours each time. These sections carry two (non-graded) credits in addition to the five (graded) credits of calculus.

4. The motivation for creating WES was to increase the success rate in calculus, and thus (presumably) the retention rate in science majors, of "underrepresented" groups, mainly minorities and women. The reasoning is that this approach offers a way for students with interest and ability in science and math to make contact with other such students, and that minorities and women, and also students from very small high schools, do not easily make these contacts on a campus as large as this one; thus WES provides the formal mechanism.

5. The WES sections have done very well academically (see attachments).

6. This fall, however, there are considerably fewer minority students than there were last fall. Last fall the classes were approximately 37% minority; this fall the classes are less than 20% minority.

7. Each WES TA has a 50% appointment for teaching one WES section. Student Assistant wages, photocopying, mailing costs, social events and field trips are about $1,600 per section per semester. The position of WES Coordinator is considered 40% of Melinda Certain's nine-month Faculty Associate position.

8. Benefits other than those to the students in WES include: a different kind of teaching experience for the TAs which involves collegiality with the other WES TAs and a lot of supervision by the Coordinator; an involvement with math and teaching by the undergraduate SAs; favorable PR for the math department (WSJ article; invited talks by Coordinator).

Melinda Certain, 10/9/97

WES DISCUSSION SECTION GPAs COMPARED TO LECTURE GPAs

Second semester, '96-'97:

221 Lecture GPA: 2.61;	WES GPA: 3.57	(1st of 9 sections; 21 students)
222 Lecture GPA: 2.43;	WES GPA: 2.85	(3rd of 13 sections; 13 students)
222 Lecture GPA: 2.87;	WES GPA: 3.25	(1st of 13 sections; 20 students)
223 Lecture GPA: 2.53;	WES GPA: 3.09	(1st of 9 sections; 11 students)

First semester, '96-'97:

221 Lecture GPA: 2.65;	WES GPA: 2.97	(2nd of 11 sections; 17 students)
221 Lecture GPA: 2.80;	WES GPA: 3.23	(1st of 16 sections; 20 students)
222 Lecture GPA: 2.51;	WES GPA: 2.78	(1st of 11 sections; 18 students)
223 Lecture GPA: 2.60;	WES GPA: 2.88	(1st of 10 sections; 16 students)

Second semester, '95-'96:

221 Lecture GPA: 2.40;	WES GPA: 2.97	(1st of 9 sections; 15 students)
222 Lecture GPA: 2.48;	WES GPA: 3.15	(1st of 8 sections; 17 students)
222 Lecture GPA: 2.55;	WES GPA: 2.68	(4th of 10 sections; 11 students)
223 Lecture GPA: 2.73;	WES GPA: 3.46	(1st of 10 sections; 12 students)

First semester, '95-'96:

221 Lecture GPA: 2.48;	WES GPA: 2.58	(4th of 11 sections; 18 students)
221 Lecture GPA: 2.56;	WES (1) GPA: 2.92	(3rd of 12 sections; 18 students)
	WES (2) GPA: 3.13	(1st of 12 sections; 18 students)
222 Lecture GPA: 2.46;	WES GPA: 3.56	(1st of 12 sections; 16 students)
223 Lecture GPA: 3.26;	WES (1) GPA: 3.54	(2nd of 10 sections; 14 students)
	WES (2) GPA: 3.86	(1st of 10 sections; 18 students)

From LEAD report by Steve Kosciuk, July, 1997

Executive Summary

During the Fall semesters from 1993 to 1996 the UW-Madison Department of Mathematics ran a total of 11 Wisconsin Emerging Scholars (WES) sections distributed over several first-semester calculus (Math 221) lectures. The students considered here were all first-semester freshman with no advanced standing, were 18 or 19 years old, and had enrolled in Math 221 in one of those four Fall semesters. Overall, we compared 169 WES students to 3,871 non-WES students.

We examined the WES program in terms of its impact on students:

1. success in calculus
2. retention in science, math, engineering, or technology (SMET) majors.

"Success" in calculus was quantified in terms of the proportions of students receiving a B or above in calculus. Specifically, for the various groups of students of interest, we analyzed the "odds of success," defined as the ratio of the number of students in the group with a B or above to the number with a BC or below. This measure was chosen because it concisely captures the most relevant part of the distribution of grades for our purposes. In contrast, differences in "mean" grades, for example, leave unanswered the question of whether one groups' higher average was due simply to more Cs in proportion to Ds, as opposed to more As and Bs in proportion to Cs and Ds. See (1,2) for some analyses in terms of mean grades.

Both of these factors were broken down by several other factors of interest. These include prior achievement or preparation (e.g., ACT, SAT math scores, UW-math placement scores, etc.), calculus lecture, gender, minority status, and whether the student was in the College of Engineering.

1. **Impact of WES on success in calculus**
   Roughly speaking, we can characterize the impact of the WES program on success rates in calculus by saying that no matter how we cut the data--by gender, minority status, engineering status, or prior achievement--the odds that WES students received a B or above in calculus were observed to be about twice that of their non-WES counterparts with a 95% confidence interval for this odds of success ratio of about (1.5, 3.0).

2. **Impact of WES on retention in SMET majors**
   The story here is also quite simple: there was no statistically significant association between persistence in a SMET major or more specifically persistence in engineering, and participation in the WES Math 221 program. That is, retention rates for the various groups were about the same for the WES participants as their non-WES counterparts. In fact, for some groups the retention percentages for WES were actually lower, although not statistically significantly so. Although another study of UW-Madison freshman enrolled in both math and chemistry in their first semester indicated that success in calculus is strongly correlated with persistence in SMET majors (6), if we use 1st semester enrollment in the College of Engineering as a proxy for enrollment in chemistry we find that our WES sample is too small (57 students) to say anything conclusive. In other words, even statistically "significant" (or insignificant) differences should not be interpreted as providing definitive conclusions when only a handful of students is in question. In fact, conclusions drawn from the total WES sample of 169 should also be treated with caution. See Appendix B.

Compiled by Student Academic Affairs, L&S

MATH 221
Fall 1995

Group	Number	Average 221 Grade	Drop/Withdraw	Non-WES
All Targeted Minorities in 221	63	2.21	15	2.142 GPA (45)
All Targeted Minorities in WES 221	18	2.38	5	
All African-Americans in 221	15	2.27	4	1.97 GPA (10)
All African-Americans in WES 221	5	2.83	2	
All Hispanics in 221	27	2.02	6	1.99 GPA (19)
All Hispanics in WES 221	8	2.08	2	
All Southeast Asians in 221	15	2.08	2	2.10 GPA (12)
All Southeast Asians in WES 221	3	2.00	0	
All Native Americans in 221	6	3.83	3	2.245 GPA (4)
All Native Americans in WES 221	2	4.00	1	

MATH 222
Spring 1996

Group	Number	Average 222 Grade	Drop/Withdraw	Non-WES
All Targeted Minorities in 222	40	2.28	8	2.044 GPA (27)
All Targeted Minorities in WES 222	13	2.77	0	
All African-Americans in 222	12	2.55	2	2.18 GPA (6)
All African-Americans in WES 222	6	2.92	0	
All Hispanics in 222	14	2.00	3	2.09 GPA (11)
All Hispanics in WES 222	3	1.67	0	
All Southeast Asians in 222	12	2.11	3	1.757 GPA (9)
All Southeast Asians in WES 222	3	3.17	0	
All Native Americans in 222	2	3.25	0	2.50 GPA (1)
All Native Americans in WES 222	1	4.00	0	

AVERAGE SIZE OF WES DISCUSSION SECTIONS

		WES Sections			Non-WES Sections		
Semester	Course	Number	Avg Size at 2nd Week	Avg Drops by End of Semester	Number	Avg Size at 2nd Week	Avg Drops by End of Semester
Fall 1995	Math 221	3	19.3	1.3	54	24.2	1.8
	Math 222	1	16.0	0.0	38	21.9	2.6
	Math 223	2	16.0	0.0	24	23.6	1.4
Spring 1996	Math 221	1	15.0	0.0	29	20.3	2.4
	Math 222	2	14.0	0.0	41	21.0	2.1
	Math 223	1	12.0	0.0	23	21.2	1.8
TOTAL		10	16.1	0.4	209	22.2	2.1

Sources: DIR and the report on "Percentage Distribution of Grades"
UW-Madison Office of Budget, Planning & Analysis
bdb 06-Dec-96