Report

of the

1995 - 1996

Mathematics Department

Assessment Committee

The Assessment Committee of the Mathematics Department was appointed, and proceeded to function, during the year pursuant to the department's adoption of the department's plan for assessing its undergraduate major. A copy of the plan is enclosed with this report.

In part the assessment plan calls for independent grading of problems selected from final exams of courses to be chosen from the central core of the department's program. This year the committee decided to concentrate on undergraduate offerings in the area of algebra as a vehicle to assess the effectiveness of the program. The courses chosen for assessment were Math 340 and Math 475 taught by Georgia Benkart, and Math 541 taught by Donald Passman. Georgia and Don are noted for their teaching proficiency.

Math 340 is an introductory course in linear algebra. It is the "gateway" to upper level undergraduate courses. Performance in Math 340 is often used to give guidance to a student contemplating a major in mathematics. Along with those who may need a mathematics major, the bulk of the students enroll in this course to satisfy mathematical needs in other disciplines.

Math 475 is an introductory course in combinatorics. It is cross listed with mathematics in both the Computer Science and Statistics departments. Georgia's section was the only one offered and had a total enrollment of 24 with 14 from mathematics and 10 from computer sciences.

Math 541 is a one semester course in modern algebra. It is taken primarily by people in the Math program and Secondary Ed program. It is one of the core courses of the program for majors and is considered absolutely essential for students going onto graduate work in mathematics. It has the reputation of being one of the more demanding courses in our curriculum.

We include the description of these courses from our undergraduate guidebook.

The questions from the finals were selected after discussions by the committee with Georgia and Don. They were picked so as to be "central" to the themes of the course and to be indicative of material we would expect a major in mathematics to be able to deal with comfortably. The grading scheme was set up by the committee. Each scoring element was to be on a zero to four scale, basically A to F. The grading was done by Susan Hollingsworth during the summer of 1996. Susan is a very experienced TA, grader and graduate student in the mathematics department. We enclose a copy of the questions and the plan for grading along with the grading results and a few observations on the students' performance by the committee's chairperson.

COURSE DESCRIPTIONS

MATH 340 ELEMENTARY MATRIX AND LINEAR ALGEBRA 3CR

This course introduces the student to matrix and linear algebra which are used in many advanced math courses and courses in other departments. Math 340 also serves as a bridge between the problem solving calculus courses and the more abstract advanced math courses; it is a prerequisite for 521, 541 and many other advanced courses. Topics: Matrix algebra, systems of linear equations, determinants, vector spaces, linear independence, bases, dimension, linear transformations, eigenvalues, eigenvectors, inner product, orthogonality, diagonalization. Possible text: Elementary Linear Algebra, 6th Ed., by H. Anton; Linear Algebra and Its Applications, by D.C. Lay. Prereq: 223. Students may not receive credit for both 320 and 340.

MATH 475 INTRODUCTION TO COMBINATORICS 3CR
(Same as Stat 475 and Computer Sci 475)

This is an introduction to the ideas and techniques of combinatorics, many of which have applications to the physical, biological and social sciences, as well as in mathematics and computer science. The emphasis is on problem solving and constructive methods. Topics: pigeonhole principle, permutations and combinations, binomial coefficients, inclusion-exclusion principle, recurrence relations, systems of distinct representatives, combinatorial designs, graph theory, optimization problems. Possible texts: Introductory Combinatorics, by R. Brualdi; Introduction to Combinatorial Mathematics by C. Liu. Prereq: Math 320 or 340 or concurrent registration.

MATH 541 MODERN ALGEBRA 3CR

This is the first semester of an introduction to basic abstract algebra. It is essential for students preparing for graduate studies in mathematics or in some related fields. Topics: group theory: subgroups, homomorphisms, isomorphisms, normal subgroups, permutation groups, class equation, Sylow theorem, finite abelian groups; ring theory: homomorphisms, isomorphisms, ideals, integral domains, polynomial rings. Possible texts: Abstract Algebra by Herstein; Contemporary Abstract Algebra by Gallian. Prereq: Math 320 or 340.

Math 340 Question

10. *(15 points)*

(a) Show for any linear transformation $T\colon V \longrightarrow W$ that the range of T is a subspace of W.

(b) Suppose that $\mathbb{R}^4 \xrightarrow{T} \mathbb{R}^3$ is the linear transformation given by multiplication by

$$A = \begin{pmatrix} 0 & 2 & 1 & -2 \\ 3 & 1 & 2 & -1 \\ 2 & 0 & 1 & 0 \end{pmatrix}$$

so that $T(\underline{v}) = A\underline{v}$. Find $\ker T$.

(c) For the transformation in part (b), what is the dimension of the range of T? *Justify you answer.*

Math 340
Instructions to Grader

10.(a)

i. Know what Range (T) is.

ii. Know how to show something is a subspace.

iii. Show Range (T) is a subspace.

iv. Do they use linearity of T correctly?

(b)

i. Understand how $T(v) = A(v)$ gives a linear transformation $\mathbb{R}^4 \longrightarrow \mathbb{R}^3$.

ii. Know what $\ker(T)$ is.

iii. Can they find solution space of $Ax = 0$?

(c)

i. Understand what dim of subspace is.

ii. Can they find dim(Range T)?

iii. Rank/nullity Theorem and column space (A).

Math 475 Question

9. *(25 points)*

   (a) Suppose you have the permutation $f(1) = 4$, $f(2) = 5$, $f(3) = 2$, $f(4) = 6$, $f(5) = 3$, $f(6) = 1$. What is the inverse permutation f^{-1}? What is the power p so $f^p = \imath$?

   (b) How many permutations of $\{1, 2, 3, 4, 5, 6\}$ have exactly 2 numbers in their natural positions?

   (c) Suppose people $\{1, 2, 3, 4, 5, 6\}$ are arranged at a circular table. Person 1 refuses to sit to the left of person 2. How many ways can you seat them?

   (d) How many permutations of $\{1, 2, 3, 4, 5, 6\}$ have no even number in their proper slots?

   (e) Count the number of permutations $a_1 a_2 \ldots a_6$ of $\{1, 2, 3, 4, 5, 6\}$ for which $a_1 \neq 6$, $a_6 \neq 1$, $a_3 \neq 4$, and $a_4 \neq 3$.

Math 475
Instructions to Grader

9.(a)

i. Know what f^{-1} is and compute it.

ii. Know what f^p is and compute it.

iii. Know what $\imath$ is and find p so that $f^p = \imath$.

(b)

i. Use condition of <u>exactly</u>.

ii. Use idea of "natural" position.

iii. Count correctly.

(c)

i. Use condition of <u>circular</u>.

ii. Use condition of person 1 etc.

iii. Count correctly.

(d)

i. "proper slots".

ii. Count correctly.

(e)

i. Use restrictions "and" correctly.

ii. Count correctly.

Math 541 Question

1. Let F be a field and let $f(x)$ and $g(x)$ be nonzero polynomials in $F[x]$.

   i. Define the greatest common divisor of $f(x)$ and $g(x)$.

   ii. Suppose $F = \mathbb{Z}/7, \mathbb{Z}$ is the ring of integers modulo 7, and that $f(x) = x^2 + 6x + 1$ and $g(x) = x^3 + x + 3$. Use the Euclidean algorithm to comput $\gcd(f, g)$.

Math 541
Instructions to Grader

1.

i. Good mathematical definition of *gcd*. English useage.

ii. Can they perform Euclid Algebra in $\mathbb{Z}_7$? Can they arrive at gcd using Algebra? Do they know arithmetic over $\mathbb{Z}_7$?

PLAN FOR ASSESSING THE MATHEMATICS MAJOR

1. GOALS

The overall goal of our undergraduate mathematics major is to produce students who understand and appreciate mathematics, who can use mathematics in understanding the world, and who can use mathematics as a basis for life-long learning. Included in this overall goal is the belief that completing the major in mathematics entails gaining sufficient subject competency to enable a student to achieve at least one of the following:

a. To handle the mathematical demands of a technical entry level position in business, industry, or government.

b. To pursue a graduate program in the mathematical sciences.

c. To handle the mathematical demands in pursuing a scientific graduate or professional program.

d. To teach mathematics in a secondary school.

2. OBJECTIVES

Students completing the major in mathematics should have attained the following:

a. They should be able to use the language of mathematics both in its idiomatic and rigorous forms. They should be able to give a clear written or oral explanations of the meaning of certain fundamental concepts or statements, or of how such statements or concepts apply in a particular situation. This ability includes interpreting and using conventional mathematical notation.

b. They should have reasonable facility with the basic mathematical techniques used in a required area of study, and a knowledge of basic theorems in this area.

c. They should be able to construct simple mathematical proofs, and to formulate and test conjectures.

d. They should be able to apply what they have learned in one mathematical area to another area, whether by modeling a physical situation or interpreting one mathematical object or structure in terms of another.

3. IMPLEMENTATION

Each year the chair shall appoint an Assessment Committee which shall be charged with gathering data indicating the extent to which the department is meeting its objectives, and, where indicated, to make recommendations based on their findings. Specifically, the Assessment Committee shall

a. Each year select one or more courses central to our major program, which will be assessed. In each of these courses, the committee, together with instructors teaching the course, will identify a question which will be made part of one of the usual course examinations. Performance on this question will be used by the committee as a measure of the attainment of the department's objectives. The questions chosen will be of types normal for the courses chosen. The responses to the questions will be graded as usual by each instructor as part of the grading process. They will also be graded independently by someone hired to assist the Assessment Committee. The questions will normally be identified early in the semester by the committee and the instructors involved. The Assessment Committee will meet with participating faculty and course coordinators for the courses involved (which may include prerequisite courses) to discuss the results.

   The courses selected by the committee for this process will normally be chosen from Math 441, Math 521, or Math 541, or other such courses as the committee judges will yield useful data to measure progress toward the department's objectives.

b. From time to time, conduct exit interviews or surveys with mathematics majors who are about to graduate.

c. About every five to seven years, conduct surveys of our graduates several years after they have graduated.

d. Collect data in such other ways as the Assessment Committee shall deem helpful and appropriate.

Once each year the Assessment Committee shall prepare a report on its activities for the year, and its evaluation of the outcome. It shall meet with the Undergraduate Program Committee to present its report and to discuss possible program modifications or improvements relevant to the material in its report.

REPORT OF THE UNDERGRADUATE PROGRAM COMMITTEE ON ASSESSMENT PROCEDURES

The Board of Regents and the North Central Association have mandated that the University develop procedures for measuring and evaluating student outcomes in general education, in each undergraduate major, and in graduate education. There are three major components that must be present in any assessment plan. They are (according to a L&S document on assessment):

1. Each unit that is being assessed should articulate clearly and precisely a set of EDUCATIONAL GOALS.

2. Each unit should develop MEANS OF ASSESSMENT that measure the extent to which it achieves the articulated goals.

3. Each unit should use these measurements to MONITOR ITS PROGRAM AND MAKE CHANGES. These may be changes in the methods used to reach the goals, or may be changes in the goals themselves.

All departments and programs are to submit plans by December, 1996, to be reviewed by the College administration and the L&S Academic Planning Council. Certain department, of which Mathematics is one, have been asked to prepare preliminary pilot statements this academic year.

The attached plan for assessing the Mathematics Major has been prepared by the Undergraduate Program Committee (UPC) for consideration by the full Department at our final Department meeting for the academic year (probably on April 20). The UPC is distributing this document now so that if there are major concerns about any part of it the UPC has time to distribute a modified document in plenty of time before the Department meeting. So if you have any concerns, please express them as soon as possible to a member of the UPC.

Assessment is a continuous activity with ongoing changes. In particular, this plan should not be regarded as the last word. It will perhaps need modification after we have gained experience with assessment.

Grader

Comments

and

Scores

Math 340

10a Only about a third of the students seemed to have a good understanding of what the range of a linear transformation is. The rest either do not know what it means, or do not think it is relevant to the problem. See Table 1.

Too many of the students seemed to have no idea what a subspace is. The others know that a subspace must be closed under addition and scalar multiplication, but only one student thought to mention that the range of T is not the empty set. I'm feeling pedantic today, so everybody who neglected this got docked. See Table 2.

A number of students used the linearity of T to show that the target space, rather than the image, was closed under addition and scalar multiplication. A goodly number didn't even try to use linearity, and failed utterly to show the range was a subspace. See Table 3.

10b All but two of the students began by row-reducing the augmented matrix, from which we may infer that they correctly understood the relationship between ker$T$ and the null space of A, and also that they have some understanding of what the kernel is.

Most students were able to solve the problem correctly. The majority wrote the answer as

$$\ker T = s \begin{bmatrix} -1/2 \\ -1/2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

which seems like sloppy notation to me, but I gave full credit for it anyway. See Table 4.

10c Nine students soleve the problem correctly using the rank-nullity theorem.

Six students solved the problem correctly using the dimension of the column space.

Seven students said that the dimension of the range was 3 because the transformation was $R^4 \longrightarrow R^3$.

See Table 5.

Math 475

9a Almost all the students were correctly able to find f^{-1}. From this we may infer that they "know what f^{-1} is" insofar as they know it to be either "that permuation which sends everything back where it came from" (most seem to have thought of it this way) or "the permutation obtained by reversing the cycles in the cycle structure" (a small minority did it thus). It cannot be determined from this problem whether they also understand that $f \circ f^{-1} = \text{id}$. See Table 6.

Almost nobody actually computed powers of f to solve the second part of this problem. Most showed the cycle structure and took the LCM of the cycle lengths. A few showed no work at all.

I have coded as follows: the grade appears as 3 digits x–yz

x in range 0 to 4 — could they find p?

y a binary digit — is it clear that they know what powers of f are?

z a binary digit — is it clear that they know what i is?

(a mark of zero in y or z simply indicates insufficient data to make a judgement).

See Table 7.

9b Again, the vast majority of students answered this problem correctly. The students' use of the notation "D_4" for "number of derangements on 4 letters" indicated an understanding of the condition of *exactly* two letters fixed, even when no explanation was provided.

I added a fourth category, graded on a binary scale, to indicate whether any work was shown (only about half showed any work at all).

See Table 8.

9c Most students counted correctly. About half did it by counting the legal configurations, and half by counting the illegal configurations.

Of the errors, one person didn't know how to use the circular condition, once misinterpreted "to the left of" as "next to", and one person apparently counted 4 arrangements but wrote 4! (out of habit?).

See Table 9.

9d More than three quarters of the students solved the problem correctly.

It appears that most of those who made a mistake interpreted the problem as, how many permutations map odd numbers to odd numbers and even numbers to even numbers and map no even number to itself?

See Table 10.

9e Almost all the students solved this problem correctly. See Table 11.

Math 541

1i A lot of students provided extraneous information in their definitions.

Quit a lot said the GCD was "the largest" or "the greatest" common divisor, without explaining what this might mean.

Three of them said $d(x) \in F[x]$ is a GCD if $d(x)|f(x)$, $d(x)|g(x)$ and *there is a* $c(x) \in F[x]$ with $c(x)|f(x)$, $c(x)|g(x)$ and $c(x)|d(x)$.

Overall, the students' use of English was not impressive.

See Table 12.

1ii Five of the students could perform the Euclidean algorithm, but misinterpreted the results and put down the wrong GCD. A few performed everything correctly except for mis-copying the problem in the beginning. See Table 13.

Table 1: Subscores for problem 10a, part i

score	occurrences
4	12
3	2
2	8
1	2
0	10

Table 2: Subscores for problem 10a, part ii

score	occurrences
4	1
3	25
2	1
0	7

Table 3: Subscores for problem 10a, parts iii and iv

score	occurrences
0,0	17
0,2	2
0,4	3
1,0	1
1,1	1
2,4	8
3,4	2

Table 4: Totals for problem 10b

score	occurrences
(illegible)	1
4,4,4	25
4,4,3	2
4,4,1	1
4,3,3	1
4,2,2	2
1,1,1	1
1,0,0	1

Table 5: Totals for problem 10c

score	occurrences
(illegible)	3
4,4,RN	9
1,0,RN	1
4,4,CS	6
4,2,CS	1
4,1,×	2
4,0,×	1
2,0,×	7
1,0,×	2
0,0,×	2

Table 6: Subscores for problem 9a, part i

score	occurrences
(illegible)	2
4	22
0	1

Table 7: Subscores for problem 9a, parts ii and iii

score	occurrences
(illegible)	2
0—00	2
1—00	1
4—00	14
4—01	2
4—11	4

Table 8: Totals for problem 9b

score	occurrences
4,4,4,0	12
4,4,4,1	10
0,4,2,0	1
4,4,2,1	1
3,2,1,0	1

Table 9: Totals for problem 9c

score	occurrences
(illegible)	1
4,2,3	1
1,4,2	1
4,4,2	1
4,4,4	21

Table 10: Totals for problem 9d

score	occurrences
(illegible)	3
4,4	17
2,1	4
4,1	1

Table 11: Totals for problem 9e

score	occurrences
(illegible)	2
4,4	20
4,2	2
0,1	1

Table 12: Totals for problem 1i

score	occurrences	score	occurrences
0,0	2	2,2	1
0,2	2	2,3	2
0,3	1	3,1	2
1,0	3	3,2	2
1,1	1	3,3	1
1,2	1	3,4	1
1,3	1	4,2	2
2,0	2	4,3	4
2,1	1	4,4	2

Table 13: Totals for problem 1ii

score	occurrences
0,0,0	4
1,0,0	1
1,0,4	1
1,1,2	1
3,1,4	1
3,2,2	1
3,4,4	2
4,1,4	5
4,4,4	15

SOME GENERAL OBSERVATIONS
BY THE COMMITTEE CHAIR

Student answers to the problems chosen as assessment vehicles seem to confirm impressions widely held by the mathematics faculty regarding the learning experience of students in the assessed courses.

Math 340 - Linear Algebra A large majority of the students enrolled in this course do not intend a major in mathematics. No attempt was made to single out for assessment the performance of intended mathematics majors. For almost all students in this course it is a first exposure to "abstract" mathematics. As evidenced in the performance on the assessment problem, some important concepts elude all too many students. Other concepts, more closely tied to calculational examples, seem to be grasped in part at least - by most of the students. Given the heterogeneity of the clientele served by this course, and their non-theoretically based calculus background, their difficulty with generalization and abstraction may be expected. Since a course in linear algebra can take many forms, it is likely that for many who enroll in it, our current version of Math 340 does not optimally suit their future needs. The department has begun to address this concern with the reinstatement of its Math 320 course which, with its emphasis on applications to differential equations, seems more useful for most engineering students.

Math 475 - Combinatorics The sample assessment in this course indicates that this course is successful. Students do seem to understand the underlying ideas and are able to apply these ideas to concrete problems. Unlike Math 340, the abstraction level of this course is not high and the concepts are more closely tied to the applications.

Math 541 - Algebra The problem selected here for assessment was adequate to assess the students' ability to write a definition and to solve a fairly simple problem in the framework of a concrete realization of an abstract algebraic structure. Many students even at this level seem still incapable of writing a clear concise definition. Most were able to solve the problem at hand. In retrospect, the problem selected here does not give us any idea of the student's ability to present a clear proof of a mathematical statement. The committee will certainly choose at least one problem in its next assessment effort to evaluate this important aspect of a math major's competence.

This report will be presented to the Mathematics faculty at its next departmental meeting. The results will be discussed with Professors Benkart and Passman and with the faculty coordinators of Math 340, 475, and 541.